A NEW MODEL FOR PERFORMANCE PREDICTION OF HARD ROCK TBMS.

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ABSTRACT

A new theoretical/empirical model has been developed for performance prediction of hard rock TBMs. The model uses information on the rock properties and cutting geometry to calculate TBM rate of penetration. The model can also be used for TBM cutterhead design optimization to achieve maximum performance in a given geology.

INTRODUCTION

There have been numerous efforts in last two decades to develop methods to accurately predict the penetration rate of a TBM in a given geology. These models are mainly based on some theoretical analysis combined with empirical data. In general, the models can be divided into two main groups, fully empirical, and theoretical/empirical. The first group is based primarily on data collected in the field and is merely a regression between machine parameters, rock properties, and the penetration rate. A good example of this is the Norwegian (NTH) hard rock diagnostic system and predictor. The other group goes into more detail of the rock cutting process by estimating cutting forces acting on individual cutters to achieve a certain penetration. These models are based on theoretical analysis of the rock fragmentation process with mechanical tools. They are supported by data generated from various types of laboratory tests such as linear cutting test, punch penetration test etc. This group includes the CSM predictor model and some of the models developed by the manufacturers for estimating TBM penetration rate. This group of models

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has the advantage of being applicable for use in cutterhead design and optimization. The cutting forces provided by these models can be used either for estimating machine thrust, torque, and power requirements, or for a given machine, they can be used to develop estimates of penetration rate.

In this paper, the basic principles of rock cutting with disc cutters, especially Constant Cross Section (CCS) cutters, are discussed and a theoretical model developed on this basis is introduced to provide an estimate of disc cutting forces as a function of rock properties and the cutting geometry. The use of the new model for TBM cutterhead design will also be discussed.

BACKGROUND

Since the introduction of mechanical excavation technology, there have been numerous studies to explain the interaction between rock and mechanical cutting tools. From the early stages, it was realized that the cutting geometry has a significant effect on the cutting forces. For disc cutters, it has been shown that the cutting forces increase with increasing cut spacing and depth of penetration. Also, there is an optimum spacing to penetration ratio which provides lowest specific cutting energy, meaning most efficient performance (Ozdemir et al 1978). Optimizing the spacing between cutters has been the common practice in industry for designing cutterheads without going into a detail analysis of the cutting process in different rock types.

In spite of all the studies completed on this subject, so far an extensive model of the cutting process that can explain all the observed phenomena has not been developed. There is still considerable debate about the mode of failure in rock cutting with mechanical tools especially disc cutters. Radial cracks and some other observations support the occurrence of tensile failure while the presence of shear forces and some shear faces in chips formed suggest shear failure. Different models based on one of these failure modes have a limited application depending on rock type and cutting tool geometry. One of the models using the shear failure was the original CSM predictor model developed for performance prediction of V–shape disc cutters. This model has been used in many cases with results close to those obtained in tests and in the field. Another example is the model developed in Sweden based on chip formation due to bending of material between two cuts (Lindqvist 1982). Obviously these models are inapplicable for CCS disc cutters that have now become the standard cutting tool on present day hard rock TBMs. Predictor models have also been derived based on measurement of force–penetration behavior using an indentation test. In this method the cutting forces are estimated based on a critical force required to penetrate a certain distance into the rock (regardless of failure mode etc.).
The new CSM model introduced in this paper is developed to estimate the cutting force requirement of CCS disc cutters at a given cut spacing and penetration in a rock of known properties. The results generated by this model can be used in TBM performance prediction and also for optimizing cutter layout and geometry to achieve maximum performance.

**BASIC ASSUMPTIONS**

As with any modelling effort, certain assumptions are applied regarding the mathematical modeling of the CCS cutters. The observations made by nearly all the researchers have confirmed the existence of a crushed zone or the so-called pressure bulb under a disc cutter as it penetrates the rock. This zone provides the means for transfer of stresses into the rock medium (Fig. 1.). The exact configuration of this zone is not known, but for the purposes of simplicity, it is assumed to be circular. This zone consists of some fine grained crushed rock that is developed due to high stress concentrations in the area immediately under the cutter. The size of particles increases from the center towards the rock media surrounding the pressure bulb. The extension of this zone is a function of cutter tip geometry and rock properties. Reducing the size of this zone is preferred for several reasons such as decreasing the amount of dust generated, increasing the efficiency of cutting, and reducing the specific energy of cutting (finer particles take more energy of cutting).

The exact pressure distribution in this zone (in a cross section perpendicular to the cut) is not known. For the ease of calculations, a uniform (hydrostatic) pressure is assumed to exist within the crushed zone (Fig. 2.).
Radial cracks are caused by the induced stresses in this zone. These cracks are the dominant discontinuity surfaces created around the crushed zone. Hence, the tensile fracture initiation and propagation is assumed to be the principal means of chip formation, and is considered to be the major failure mode. It is noteworthy that cracks induced around the crushed zone (especially those continued to the free surface) are subject to significant shear stresses, as well. Therefore, a mixed mode failure for chip formation is closer to reality. Interaction between the cracks from two adjacent cuts guides the propagation of fractures. When one or more cracks from neighboring cuts meet or cracks reach the free surface, chipping occurs. The length of the cracks is a function of the pressure in the crushed zone, which in turn is a function of the cutter normal force. The cut interaction depends on spacing between the cuts, the angle, and extension of cracks between the adjacent cuts. According to fracture mechanics principles, a crack may propagate in any direction which provides the least surface energy and until the stress intensity is over the critical stress intensity factor of rock. This propagation can continue until stress intensity drops below the critical value or crack meets a free surface (that can be another surface). This means that chips can be formed by cracks in any angle. Thus, occurrence of ridge formation and overbreak of rock under different cutter loads can be explained. In the case of ridges, cracks are developed toward cutting face and reach free surface and form small triangular chips. This will cause a drop in the pressure of crushed zone and prevents further propagation of cracks toward other cuts and leaves the material between the cuts almost untouched (Fig 3.a.). The process may continue until in subsequent passes with higher cutter loads when two of the cracks can interact and form a chip. If the applied load is too high, longer cracks can develop inward and meet in an angle, meaning an overbreak (Fig 3.b.). This produces thicker chips below cutting level. For an optimum spacing, cracks are ideally propagated towards the neighboring cuts through a straight line which would be the shortest distance for crack propagation and is equal to
a. Ridge formation due to lack of pressure and length of cracks

b. Over break due to excessive loading and longer cracks.

c. Normal cutting with optimum crack length and direction.

Fig. 3. Chip formation in different situations
half the spacing. (Fig. 3.c.). For modelling purposes, the later case is considered to prevail while cutting with disc cutters.

PRESSURE DISTRIBUTION AND CUTTING FORCES

In order to estimate the forces acting on a disc cutter, one should integrate force elements caused by pressure acting on disc cutter in 3D. Since a uniform crushed zone pressure around the cutter tip is assumed to exist, side forces can be neglected, as pressures on both sides of cutter tend to cancel out. In fact, this is a valid assumption since side force is caused by chip formation on one side of the cutter and the unbalanced pressure on the other side. In a longitudinal cross section (along the cut), force elements can be integrated to obtain normal and rolling forces. For this purpose, a pressure distribution along the periphery of the disc cutter (within rock-cutter interacting area) as shown in Fig 4. is assumed to exist. This region of interaction can be specified by angle $\phi$ that is determined from cutter diameter and penetration as follows.

$$\phi = \cos^{-1}\left(\frac{R - p}{p}\right)$$ (1)

Where $R = \text{cutter radius or D/2}$
$p = \text{penetration}$

Magnitude of pressure, $P$, at any point is a function of the angle $\theta$ and the base pressure $P'$. This function can be written as:

$$P = P'\left(1 - \frac{\theta}{\phi}\right)^\psi$$ (2)

Where $P' = \text{Base pressure}$
$\theta = \text{Angle from the normal to face, ranging from 0 to } \phi$.

This function can generate different pressure distributions as power $\psi$ changes. For a linear distribution starting from zero in front of the cutter and maxing to $P'$ under the cutter in a linear fashion with $\theta$, $\psi=1$ is used (Fig.4.a.). A value of $\psi=0$ will generate a uniform or constant pressure distribution along the cutter penetration edge. In general, increasing $\psi$ will shift the pressure towards the normal line (vertical cutter axis) while decreasing it will shift the pressure forward. A negative value for $\psi$ relates to higher pressures in front of the cutter. This shift in pressure in turn reflects the shift in position and angle of the resultant force $F_r$. In other words, decreased $\psi$ will increase ratio of the rolling to normal forces, which is referred to as the Cutting Coefficient (CC) or Drag Factor. Normal and rolling forces, $F_n$ and $F_r$, are components of the resultant force projected on the X and Y axes (see Fig.4.). The angle of resultant force with normal to the cutting face
Fig. 4. Pressure distribution along the disc cutter periphery.

a. Linear pressure distribution

b. General shape of pressure distribution with power function.
(or Y axis), \( \beta \), can be determined by estimating cutting coefficient as follows:

\[
\beta = \tan^{-1}\left(\frac{F_r}{F_n}\right) = \tan^{-1}(CC)
\]  

(3)

Values of \( F_n \) and \( F_r \) for the linear distribution, \( \psi = 1 \), can be estimated as (Fig. 4.a.):

\[
dF = TPRd\theta = TRP' \left(1 - \frac{\theta}{\phi}\right) d\theta
\]  

(4)

\[
F_n = F_y = \int_{0}^{\phi} dF_y = \int_0^\phi dF \cos\theta = \frac{TRP'}{\phi} (1 - \cos\phi)
\]  

(5)

and

\[
F_r = F_x = \int_{0}^{\phi} dF_x = \int_0^\phi dF \sin\theta = \frac{TRP'}{\phi} (\phi - \sin\phi)
\]  

(6)

Where \( T = \) Cutter tip width

Hence the cutting coefficient for the linear distribution can be estimated by:

\[
CC = \frac{\phi - \sin\phi}{1 - \cos\phi}
\]  

(7)

In order to determine cutting forces and CC in a general case with the power function, a new coordinate system is used to simplify calculations as in Fig. 4.b. Using a new positioning angle \( \alpha \), the pressure function changes to:

\[
P = P' \left(\frac{\alpha}{\phi}\right)^\psi
\]  

(8)

Thus force components in the new system will be determined as:

\[
dF = TPRd\alpha = TRP' \left(\frac{\alpha}{\phi}\right)^\psi d\alpha = \frac{TRP'}{\phi} t^\psi dt
\]  

(9)

Where \( t = a \) slack variable replacing \( \frac{\alpha}{\phi} \)

\[
F_y' = \int_{0}^{\phi} dF_y' = \int_{0}^{\phi} dFSin\alpha
\]  

(10)

and
Since the above integrals do not have a closed or analytical solution, a numerical approach based on Taylor series development of Sin and Cos is used to determine the forces. The results of the integration are:

\[ F_{x'} = \sum_{i=1}^{n} (-1)^{i-1} \frac{\phi^{2i+\psi}}{(2i+\psi)(2i-2)!} \]

and

\[ F_{y'} = \sum_{0}^{n} (-1)^{i-1} \frac{\phi^{2i-1+\psi}}{(2i-1+\psi)(2i-2)!} \]

Here, "n" is the number of iterations needed to get the desired degree of accuracy in numerical estimate (usually \( n \geq 5 \) is enough). The angle of resultant force, \( \gamma \), is

\[ \gamma = \tan^{-1} \left( \frac{F_{y'}}{F_{x'}} \right) \]

This angle can be related to \( \beta \) in normal coordinates and \( CC \) as:

\[ CC = \tan\beta = \tan(\phi - \gamma) \]

Figure 5. shows the correlation between numerical solution and analytical values of \( CC \) for different penetrations using a 43.5 cm (17 in) cutter.

The magnitude of cutting coefficient depends on \( \phi \) and pressure distribution and has been measured for cutting different rock types with different cutter geometries. The results of modeling can be compared with measured values to validate the type of pressure distributions selected for each case. A linear pressure distribution has been used in most modelling efforts in the past but, the actual cutting data shows that a uniform distribution is closer to reality. In general value of \( \psi \) depends on the shape of cutter tip. For V–shaped disc cutters, this value increases to around 1. For CCS and blunt (or worn) cutters, \( \psi \) is closer to zero and decreases with increasing tip width. A nominal value of \( \psi = 0 \) can be used for most cases in which CCS cutters with a tip width of about 12.5 mm (0.5 in) is used. Using this nominal value, the following is obtained:

\[ \beta = \frac{\phi}{2} \quad \text{and} \quad CC = \tan\left( \frac{\phi}{2} \right) \]
Fig. 5. Correlation between numerical and analytical solutions of CC.

Table 1. can be used to estimate CC for different values of \( \psi \) for a 43.5 cm (17 in) disc cutter. For other values of \( \psi \) and cutter diameters, the value of CC can be determined using the above equations.

The cutting forces, \( F_n \) and \( F_r \), are determined by using the total force \( F_t \) and CC or \( \beta \):

\[
F_t = \frac{P'RT\phi}{\psi + 1} \quad (17)
\]

\[
F_n = F_t \cos \beta \quad (18)
\]

and

\[
F_r = F_t \sin \beta \quad (19)
\]

In these equations, all the variables are known except for \( P' \), which is a function of the cutting geometry and rock properties. A general solution for crushed zone pressures high enough to propagate cracks to form chips between two adjacent cuts (with a certain spacing in a given rock type) does not exist. However, a correlation between measured cutting forces and cutting parameters can be utilized to develop an estimate of the base pressure \( P' \). For this purpose, a database of measured forces from linear cutting tests performed in the laboratory (LCM) in various rock types and using different cutting geometries has been used. This database includes estimated pressure (from measured forces), cut spacing and penetration, cutter diameter and tip width, and the uniaxial and tensile strengths of the rock. The regression analysis of these data yields the following equations for estimating \( P' \) (psi).
Table 1. Calculated Cutting coefficient for different values of $\psi$ and penetration using a 43.5 cm (17 in) CCS disc cutter.

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<td>0.276</td>
<td>0.263</td>
<td>0.252</td>
<td>0.241</td>
<td>0.232</td>
<td>0.223</td>
<td>0.215</td>
<td>0.207</td>
<td>0.200</td>
</tr>
<tr>
<td>3.8</td>
<td>0.311</td>
<td>0.295</td>
<td>0.281</td>
<td>0.268</td>
<td>0.257</td>
<td>0.246</td>
<td>0.236</td>
<td>0.227</td>
<td>0.219</td>
<td>0.211</td>
<td>0.204</td>
</tr>
</tbody>
</table>
\[ P' = -32628 + 521\sigma_c^{0.5} \quad (R^2 = 52.5\%) \]

\[ P' = 103400 + 4200S - 7.37\sigma_t + 2.48\sigma_c - 1260p - 21030T - 11740R \quad (R^2 = 78\%) \]

\[ P' = 100500 + 12170S + 7.88\sigma_c - 28830\sigma_t^{0.1} - 192S^3 - 0.000147\sigma_c^2 - 29450T - 13000R \quad (R^2 = 86.5\%) \]

Where
- \( \sigma_c \) = Uniaxial Compressive Strength (UCS) of rock (psi)
- \( \sigma_t \) = Brazilian tensile strength of rock (psi)
- \( S \) = Spacing between the cuts (in)
- \( T \) = Cutter tip thickness (in)
- \( R \) = Cutter radius (in)
- \( p \) = Penetration (in)

\( (R^2 \) is the correlation factor for each equation)

These equations can be used depending on the number of parameters available and the degree of accuracy desired. It is important to note that there is a range of application for these equations according to the range of data used for the original regression analysis. In this case the range of application for different variables is as follows:

- Uniaxial compressive strength (UCS); 70–200 Mpa (10000–30000 Kpsi)
- Brazilian tensile splitting strength; 4–18 Mpa (500–2500 psi)
- Cutter radius; 39–45 cm (15–18 in)
- Cut spacing; 5–15 cm (2–4 in)
- Depth of penetrations; 0.25–3 cm (0.1–1.5 in)

Estimation of cutting forces for V–shape disc cutters or worn cutters can be accomplished by using an approximate value for tip width \( T' \), as follows:

\[ T' = T + w\tan\left(\frac{\alpha}{2}\right) \quad (20) \]

Where
- \( T' \) = Tip width for V–shape or worn cutter to be used in equations
- \( T \) = Tip width of sharp cutter or 0 for V shape
- \( w \) = Tip lost (worn out from radius)
- \( \alpha \) = Tip angle, 5–10 for CCS and 90–120 for V–shape cutters.

These equations yield a pressure value in psi and using other parameters, cutting forces can be estimated in lbs.

Note: The regression equations listed above are continually being updated as more cutting data becomes available to extend their range of application to a wider range of rock strength.
Having estimated the cutter forces for a set of parameters, the results can be used for TBM cutterhead design and optimization. For a given machine diameter, the number of cutters on the face, \( N \), can be estimated by dividing the cutterhead radius by the average spacing of cuts. This number must be increased by adding several cutters to account for center and gage cutters. Machine thrust is simply a multiplication of normal force by the number of cutters. The torque caused by each cutter is the rolling force times the distance of that cutter from the center of the cutterhead (radius of circle on which the cutter travels). Summation of these torque values is the cutterhead torque requirement. The rotational speed of the machine is limited by the maximum allowable linear velocity limit of gage cutters. This limit is set by cutter manufacturers to prevent excessive heat generation and potential premature damage to seals. The machine power requirements are determined by using the cutterhead torque and the rotational speed (RPM). The installed power is then calculated by considering the efficiency of the electrical and mechanical components of the machine.

All of these machine parameters are calculated based on predicted forces acting on cutters for a given set of cutting parameters and rock properties. A more detailed study of machine specifications and optimization of cutterhead layout is possible by changing the spacing and location of cutters on the face. This can be done in a systematic or on a trial and error basis to check all the governing parameters and optimizing them.

**PERFORMANCE PREDICTION**

For performance prediction purposes, the model can be used to estimate the penetration rate attainable for a given TBM. Using the information on the machine specifications and the rock properties and discontinuities, the achievable rate of penetration can be predicted. The advance rate can then be calculated using an estimate of the machine utilization factor in the ground conditions to be encountered. Machine utilization is the percent of time when the machine is excavating out of the total project time. It basically depends on the type of operation (e.g. tunnel size, grade, curves, etc.), management, maintenance of machine, and capacity of the back up system. Lower range of utilization figures reflects excavating curves and slopes, operating in severe ground conditions, poor management and/or frequent machine break down. On the other hand, high utilization is generally achieved in boring relatively straight tunnels with slight up slope in favorable ground conditions coupled with good management and maintenance practices. Down time of the machine includes regripping time, time for cutter changes and scheduled or unscheduled maintenance, halt in operation due to support installation or transportation problems, power loss, shift changes and lunch breaks, labor and miscellaneous delays etc. One must take all these parameters into account to provide a realistic utilization factor.
and advance rate.

Following section contains a working example of this model first for cutterhead design and then for machine performance prediction.

EXAMPLE OF MODEL PREDICTIONS

The following example is provided to illustrate the use of the model. Having some initial information about the tunneling operation and the ground conditions, the model can be used for cutterhead layout and estimating machine parameters. For cutting a rock with UCS of 105 Mpa (15000 Psi), and 9 Mpa (1300 psi) tensile strength with a 43.5 cm (17 in) diameter cutter with tip width of 1.25 cm (0.5 in) and spacing and penetration of 7.5 cm (3 in) and 0.5 cm (0.2 in) respectively, the base pressure of crushed zone is estimated to be:

\[ P' = 30,100 \text{ psi} = 210 \text{ Mpa} \]

Thus, the forces using \( y = 0 \) will be (Note: \( \phi = 12.5^\circ \) and \( \beta = \phi/2 = 6.25^\circ \), use equivalent in radiant)

- \( F_t = 27,620 \text{ lbs} = 123 \text{ kN} \)
- \( F_n = 27,450 \text{ lbs} = 121 \text{ kN} \)
- \( F_r = 3,000 \text{ lbs} = 13.3 \text{ kN} \)
- \( CC = 0.11 = 11\% \)

Assuming a machine diameter of 3.45m (11.5 ft) the total number of cutters will be

\[ N = \frac{3.45 \times 100}{(7.5 \times 2)} = 23 \text{ (use 28)} \]

Thus the machines thrust, torque, and power requirement using a rpm of 12 will be (since machine diameter is small, rotational speed is not restricted by linear velocity of cutters):

- Thrust = 28 \( F_n = 770 \text{ klbs} = 3.4 \text{ MN} \)
- Torque = 28 \( F_r \times 3.45 \times 0.6/2 = 290 \text{ klbs-ft} = 385 \text{ kN-m} \)
- C.head Power = torque \( \times \) rpm \( / \) Conv. Factor = \( 660 \text{ hp} \sim 500 \text{ kW} \)
- Installed power = C.head power \( / \) 0.85 = \( 776 \text{ hp} \sim 585 \text{ kW} \)

The advance rate of the machine assuming a utilization factor of 55% becomes:

\[ \text{Advance rate} = 12 \times 0.5 \times 55\% = 3.3 \text{ cm/min} \sim 2 \text{ m/hr} \]
It should be noted that these numbers are rough estimates of machine specifications, since the calculations assume even cutter loading across the face. In reality, cutters experience different levels of load depending on their position and orientation on the cutterhead. A detailed study of the cutterhead includes calculating cutting forces for individual cutters on the cutterhead. This is possible by computer programs modeling the cutterhead and determining individual cutter forces. For cutterhead design optimization, a similar approach can be used, in which by changing position of cutters, an optimum layout to achieve a balanced cutterhead with maximum penetration rate can be found. A computer model to serve this purpose has also been developed.

For performance predictions, however, the cutterhead layout is more likely fixed and the machine capabilities are defined, hence the only variable is the penetration rate. In order to determine the penetration rate of the machine using the model developed, cutterhead layout is entered into the computer program to calculate maximum attainable rate of penetration either by reaching thrust or torque/power limit of the machine. Proceeding with our example, assume an existing machine with the following specifications,

Machine thrust = 1,2 Mlbs = 5.3 MN
Machine Torque = 353 klbs–ft = 478 kN–m
Machine power = 720 C.head hp ~ 540 kW

The penetration rate of the machine can be estimated by generating a performance Table (as Table 2.). The penetration rate can be estimated as a function of available machine thrust, torque, and power.

Table 2. Machine performance prediction.

<table>
<thead>
<tr>
<th>No</th>
<th>Penetration cm/rev (in/rev)</th>
<th>Advance rate (55% utilization) m/hr (ft/hr)</th>
<th>Thrust MN (klbs)</th>
<th>Torque kN-m (klbs-ft)</th>
<th>Power kW (hp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25 (0.1)</td>
<td>0.8 (2.4)</td>
<td>2.02 (455)</td>
<td>148 (110)</td>
<td>190 (250)</td>
</tr>
<tr>
<td>2</td>
<td>0.5 (0.2)</td>
<td>2.0 (6.6)</td>
<td>2.85 (643)</td>
<td>298 (220)</td>
<td>375 (500)</td>
</tr>
<tr>
<td>3</td>
<td>1.0 (0.4)</td>
<td>4.0 (13.2)</td>
<td>4.01 (905)</td>
<td>597 (440)</td>
<td>760 (1010)</td>
</tr>
</tbody>
</table>

For our example and using the above table, the maximum attainable advance rate (with utilization factor of 55%) can be estimated as:

Maximum advance rate attainable = 2.9 m/hr (0.73 cm/rev)

Note that the results shown in Table 2. are slightly different than the calculations performed for the previous part. This is due to using assuming even cutter loading in the first case, as opposed to using detailed computer modeling of the cutterhead for the latter.
The model discussed in this paper is still under development in several aspects. Work is underway to more accurately define the shape of the pressure function for cutter force estimation. The power function used in current model has been found to generate reasonably accurate estimates of disc cutting forces in a wide range of rock types. However, it is still important to find the true function which can help to further understand the real cutting process. Once the true pressure function is determined, the information can be used to evaluate various cutter edge geometries to control the shape of the pressure function. The first intent is to change the cutter tip design to shift the resultant force closer to normal and obtain lower rolling forces, meaning less torque and power requirements. Meanwhile, a determination of the real pressure distribution can assist cutter manufacturers in analyzing the state of stresses in the cutter ring to minimize stress concentrations and ring deformations.

The other aspect of future anticipated development is to modify the equations for estimating the magnitude of the base pressure. The equations mentioned in this paper are based on the cutting data obtained to date and an extensive effort is underway to extend the data base of measured forces and update the equations. Regrouping of rocks based on their characteristics (such as igneous, sedimentary, etc.) will be undertaken with the objective of further improving the accuracy of the model. Also, an analytical solution of the problem using fracture mechanics approach is under consideration for further modification and improvement of the model.

Another step is to incorporate the geological data on ground conditions (i.e. joints, fractures, bedding, groundwater etc.) into the model. As it stands now, the effect of joints or planes of weakness are not systematically accounted for in the model. It is intended to take this information, in conjunction with other parameters into consideration for accurate performance prediction of tunnel boring machines.

CONCLUSIONS

The new model developed for performance prediction of hard rock TBM has been found to provide reasonably accurate estimates of machine advance rate by determining disc cutting forces. The cutting forces can be determined for a wide range of cutting parameters and rock properties. These forces can be used for cutterhead layout design and optimization as well as calculation of machine thrust, torque, and power requirement. The model has also been incorporated into a computer program to evaluate overall machine performance and cutter cost. Future plans include further development and improvement of the model to extend its range of applications both for soft and hard rock tunneling.
REFERENCES


